A Correction Lens for Laser Doppler Velocimeter Measurements in a Cylindrical Tube

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A lens was designed using analytical ray tracing techniques to correct aberrations caused by a cylindrical tube wall when measurements are made off the plane of symmetry with a dual-beam laser Doppler velocimeter. The lens keeps the probe volume stationary when the measurement axis is rotated and reduces aberrations in the image of the probe volume on the pinhole. A planoconcave cylindrical lens with a radius of curvature of 1195 mm (47 in.) was fabricated and tested for use with a 101.6 mm (4 in.) o.d., 3.2 mm (1/8 in.) thick acrylic plastic tube with air as the flow medium. Good results were obtained out to a normalized tube radius of $r/R = 0.83$.

Nomenclature

d	= thickness of lens at vertex
f	= focal length of lens
h	= step height for sudden expansion (22.2 mm)
$k_{i,t}$	= unit vector along incident and transmitted ray
n	= refractive index
$n_{i,t}$	= refractive index of incident and transmitting media
n_l	= refractive index of lens
r	= radial position
R	= inside radius of tube
$R_{1,2}$	= radii of lens surfaces
u_n	= unit vector normal to a surface at a point
U_0	= reference velocity, 27.8 m/s
$U_{x,r}$	= axial and radial velocity components
x,y	= Cartesian coordinates

Introduction

ONE of the most common situations in fluid mechanics is flow through cylindrical tubes. When a laser Doppler velocimeter (LDV) is used to make velocity measurements in cylindrical tubes, severe optical aberrations caused by the tube wall curvature can be present. In the symmetrical case where the optical axis of the LDV intersects the tube axis, the aberrations are normally not large enough to cause serious degradation of the LDV signal. There is a position shift of the probe volume along the LDV axis due to refraction, but this can be accounted for easily. However, in nonsymmetrical configurations, the probe volume can be distorted and enlarged significantly, in addition to which there are shifts in both position and orientation. The effect is further compounded by the similar aberrations introduced as the scattered light passes through the opposite tube wall, resulting in poor definiton of the probe volume at the photomultiplier pinhole and thus poor signal quality.

Equations have been derived[1,2] to find the actual location of the LDV probe volume and its angular orientation for different situations. These are of limited practical usefulness, however, for several reasons. In the case of a one-component LDV operating in the forward scatter mode, the receiving optics will require major realignment for each change of measurement location. This realignment will require adjustments that may not be possible on most LDV systems. Additionally, the enlargement of the probe volume and its image at the pinhole are not accounted for. With a two-component system, the situation will be even worse because the two probe volumes may be spatially separated. The obvious solution to these problems is the introduction of some type of correction element to compensate for the optical effects caused by the tube. This element must reduce the effects described to an acceptable level so that reliable measurements can be made.

This investigation addresses the common situation of a cylindrical tube with air as the medium both inside and outside the tube. In this situation the difference between the refractive indices of the tube and the air is high, causing a significant degree of aberration in tubes having reasonable wall thicknesses. If a liquid is used as the flow medium, it is often possible to match the refractive indices of the liquid and the tube, thus eliminating refraction effects at the inner surface of the tube. In addition, the cylindrical tube can be immersed in a larger vessel with flat walls. Even if a perfect refractive index match cannot be made, the aberration effect will be less severe with a liquid working fluid because the refractive index difference will be less.

Definition of the Problem

The common LDV input beam geometries used to measure axial and radial velocities in a circular duct are shown in Figs. 1 and 2. These two measurements may be made simultaneously if a two-component LDV is used, or one at a time with a one-component system. The beam geometries shown are applicable to a four-beam, two-component system. For a three-beam, two-component system, one of the beams will be used for both components, but similar problems will be encountered with alignment. Measurements off the horizontal diametral plane (x axis in Figs. 1 and 2) require some type of correction for the aberrations introduced by the curved duct wall. (Aberrations still exist when measurements are made on the horizontal diametral plane, but these are usually of less significance due to the symmetry involved.)

Figure 1 illustrates the beam paths when one attempts to measure the axial velocity component (normal to the figure) at a dimensionless radius of $r/R = 0.8$. In this example, an acrylic plastic tube ($n = 1.49$) with a wall thickness 15% of the inside radius and air as the medium on both sides of the tube is shown. The incoming beams are aligned so that the virtual intersection point is on the vertical (y) axis and the beam intersection half-angle is 7.5 deg. For this case, the probe volume is displaced so that the actual measurement

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point is as shown. The probe volume is also rotated about the axial direction, but a true axial component is still being measured. When forward scatter receiving optics are used, a cumbersome adjustment of both the transmitting and receiving optics is required for each consecutive measurement point on the y axis.

The beam paths for a radial velocity measurement at the same point are shown in Fig. 2. In this case, the probe volume is more severely displaced, the beam intersection angle is altered, the beam intersection is on the opposite side of the y axis from that of Fig. 1, and the orientation of the probe volume is changed due to the angular rotation of the beam bisector. Probe volume orientation is defined here by the normal to the fringe planes in the probe volume, which will also be perpendicular to the bisector of the intersecting beams. In this case the combined rotation and translation of the probe volume mean that a true radial component is not measured, but the two effects tend to counteract each other and the error will be small unless a significant tangential velocity exists. Obviously, however, it would not be possible to measure axial and radial components simultaneously with a four-beam system, since the probe volumes for the two components would be separated in the x direction by a substantial amount.

When measuring either an axial or a radial component, these are the only significant effects as far as the input beams are concerned. However, if the input beams lie in some other plane, the possibility also exits that they will no longer cross or, if they do, that they may be oriented at some different angle. Measurements at other beam orientation angles are frequently necessary to determine higher-order quantities such as turbulent shear stress and turbulence kinetic energy. For example, with the input beam plane at 45 deg with respect to the tube axis in the above system, the beams fail to cross. At their closest point, the beams are separated by a distance that is much larger than the beam diameter.

When the receiving optics are considered, the situation is somewhat different. The function of the receiving optics is to collect light scattered from particles in the probe volume and focus this light within the aperture (pinhole) in front of the detector. Essentially, this is the classical problem of imaging a point source, complicated in the present case by the presence of the curved tube wall in the optical path. From the standpoint of geometrical optics, the problem is one of bringing to intersection within an acceptably small region all light rays leaving the particle that fall within the aperture of the receiving lens. This is not a trivial problem, particularly when the axis of the receiving optics does not coincide with the tube axis and, in fact, must move with respect to the tube axis to permit the desired measurements.

Lens Design and Fabrication

The most important design criterion for the correction lens is that the beam intersections must be very near the same location for all input beam orientations relative to the tube axis. The different beam orientations would be due to a rotation of the beams in a one-component system or the two different beam pairs present in a two-component system. The location of the probe volume is of secondary importance as long as it remains stationary with beam plane rotation. The actual location can be easily calculated.

The concept of analytical ray tracing, the method used in the design of the lens, is straightforward. An incoming ray is directed at the first surface of the optical system and the ray intersection point at the surface is calculated. The refraction equation

$$n_i(k_i \times u_n) = n_t(k_t \times u_n) \tag{1}$$

is then applied to determine the new direction of the transmitted ray. The intersection of the transmitted ray and the second surface is then calculated and the refraction equation applied again. This process is repeated as necessary. After all of the appropriate rays have been traced through the system, the point at which the rays intersect can be calculated. In the case of the input beams, this involves finding the intersection of pairs of rays lying in planes at the various angles to the tube axis that represent the different input beam pairs required to measure the different velocity components. The optimum correction lens design is the one that causes all of these ray pairs to intersect within desired tolerances.

The tolerances chosen were that the beam intersection point remain stationary within ± 0.1 mm in the direction along the bisector of the two beams and within ± 0.025 mm in a direction perpendicular to that axis. These tolerances are approximately 10% of the corresponding dimensions of a typical LDV probe volume. Several workable lens types were found that used various combinations of cylindrical and planar surfaces. The lens design finally selected was a cylindrical planoconcave configuration. A radius of curvature of 1195 mm (47 in.) on the concave surface was determined to be optimum for the 101.6 mm (4 in.) o.d., 3.2 mm (⅛ in.) wall acrylic tube used in the present study. This lens satisfied the design requirements out to a nondimensional radius of $r/R \approx 0.8$. At larger r/R values, the angle between the incoming beams and the tube wall is very shallow and the aberrations caused by the tube are much harder to correct. A schematic diagram of the LDV system with the correction lenses is shown in Fig. 3.

Table 1 illustrates computed values for important probe volume parameters with and without the correction lens present. In both cases, the beam intersection point for the axial component measurement was on the tube centerline at

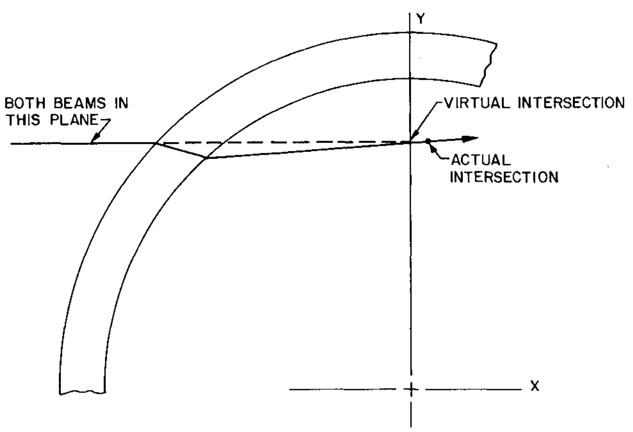

Fig. 1 Beam path for axial measurement.

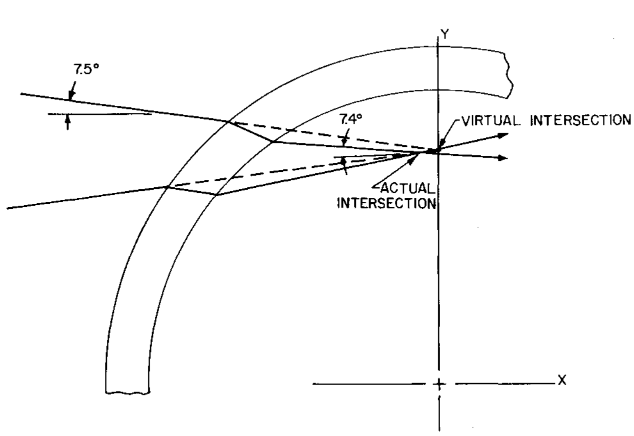

Fig. 2 Beam path for radial measurement.

Table 1 Comparison of probe volumes with and without the correction lens

Parameter		$r = 0.5$ in. $r/r = 0.27$		$r = 1.0$ in. $r/R = 0.53$		$r = 1.5$ in. $r/R = 0.80$	
		Without lens	With lens	Without lens	With lens	Without lens	With lens
Location of probe volume for axial measurement	$X =$	0.042	0.000	0.043	0.005	0.046	0.012
	$Y =$	0.500	0.511	1.001	1.026	1.503	1.556
Location of probe volume for radial measurement	$X =$	−0.003	0.000	−0.015	0.005	−0.059	0.013
	$Y =$	0.500	0.511	1.000	1.026	1.499	1.556
Probe volume orientation, deg		0.34	0.65	0.86	1.50	2.28	3.51
Actual half-angle for radial measurement, deg		3.49	3.41	3.49	3.40	3.49	3.49

$r/R = 0$. At increasing values of r/R, the coordinates of the actual beam intersection are as shown when the LDV optics are traversed in the y direction. In order to meet the design tolerances, it was necessary to adjust the correction lenses when the measurement point was moved to a different y location. The relationship between the probe volume position and the lens position is given in Fig. 4. The probe volume in this case is formed at points within ±0.75 mm (0.03 in.) of the vertical diametral line. The two lenses are moved simultaneously and are always symmetrically placed relative to the tube. A lens design that did not require this change in lens position was found to be impractical. Note that the probe volumes for the axial and radial measurements coincide with 0.025 mm (0.001 in.) when the correction lens is present. The beam intersection angle for the radial measurements does change somewhat, but this is easily accounted for in the data reduction.

The lens does not correct the rotation of the probe volume about the axial direction due to the slight angular deviation of the beam bisector, which in fact is larger when the correction lens is used. As mentioned before, this rotation has no effect on a measurement made in the axial direction, but for a measurement in the radial direction a small component of any tangential flow will be introduced. In most cases, this slight misalignment will cause no problems, but in a flow where a large swirl is present this effect could cause large errors in the radial velocity measurements.

The design of the correction lens for the receiving optics side of the system can be simplified by limiting measurements to the vertical diameter of the tube (the y axis in Fig. 1). If a correction lens identical to that on the input beam side is symmetrically placed on the receiving optics side of the tube, then light rays in a cone defined by the input beam and originating at the input beam intersection "point" can be collected and focused to a common intersection to the same order of accuracy as that achieved for the input beams. It should also be true, at least to a first approximation, that rays at angles somewhat larger and smaller intersect in the same vicinity. Figure 3 illustrates this geometry. Such a system requires only one correction lens design, an obvious cost advantage. This basic design approach was adopted for the correction lens system.

An interesting property of the correction lens was discovered after a few lens configurations were found that were workable. When the thick lens equation from geometrical optics,[3]

$$\frac{1}{f} = (n_l - 1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n_l - 1)d}{n_l R_1 R_2}\right] \qquad (2)$$

is applied to the tube wall, the effective focal length obtained is approximately the same as the effective focal length of a workable correction lens based on the same equation. This is an unexpected result because the thick lens equation is derived using paraxial theory and a small-angle assumption, which is violated for the tube wall. Various cylindrical lens configurations were checked as well as other sizes and thicknesses of tubing and the same result was found in each case, i.e., a workable correction lens has approximately the same effective focal length as the tube wall when the wall thickness is less than 20% of the inside radius. This empirical rule can be used as a starting point when designing correction lenses for new tube dimensions.

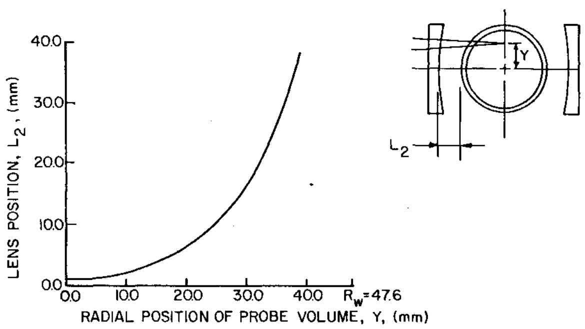

Fig. 4 Lens position required vs radial location of probe volume.

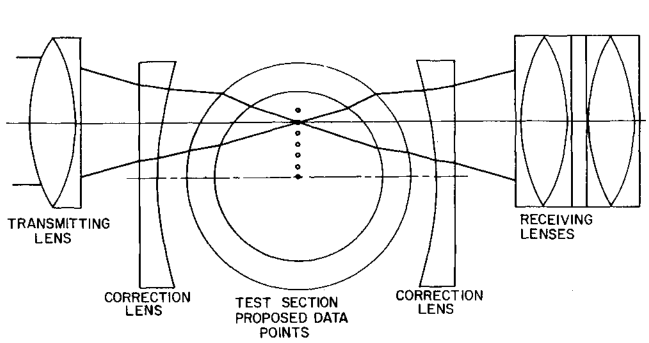

Fig. 3 Schematic diagram of correction lens inserted in LDV system.

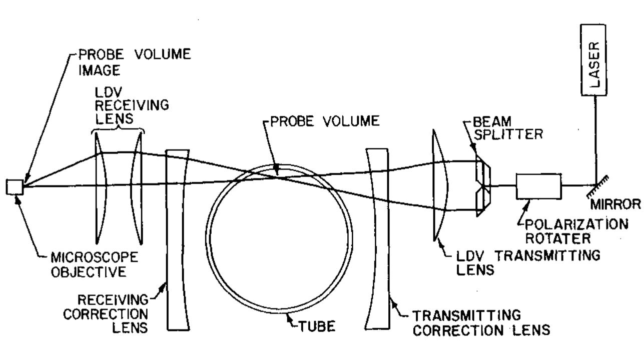

Fig. 5 Optical bench setup used to test correction lens.

Experimental Verification

The correction lens was relatively easy to fabricate, since it had only one cylindrical surface. A pair of lenses were ground and polished from acrylic plastic blocks and were tested on an optical bench to confirm that their performance would match that predicted by the ray tracing routine. A half-section of the tube was mounted on the bench with one of the correction lenses located in front of it. A microscope objective mounted on a three-dimensional position indicator was used to locate the actual beam intersection point in the tube for various beam orientations, radial locations, and correction lens positions. The measured intersection points agreed well with the ray trace predictions. The maximum deviation was approximately 0.05 mm (0.002 in.) perpendicular to the bisector and 0.15 mm (0.006 in.) along the bisector. When the plane of the input beams was rotated, the probe volume location remained stationary within the specified tolerances (0.1 mm along the bisector, 0.025 mm perpendicular to it) for all radial locations out to $r/R = 0.83$ (the actual probe volume location for a virtual location of $r/R = 0.8$). At larger radii the probe volume movement exceeded the specified tolerances when the beam orientation was rotated, but these movements were still as predicted by the ray tracing program. The lens was also effective in minimizing probe volume distortion and enlargement.

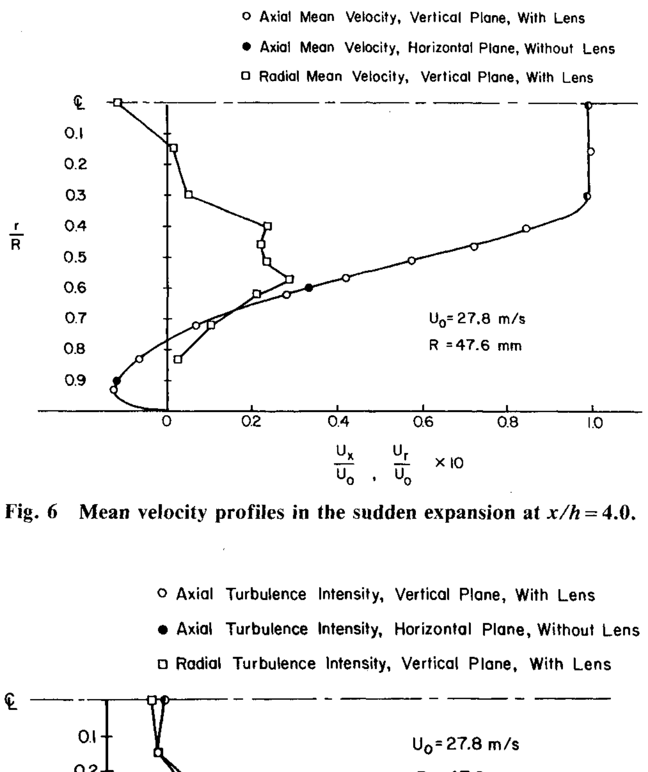

Fig. 6 Mean velocity profiles in the sudden expansion at $x/h = 4.0$.

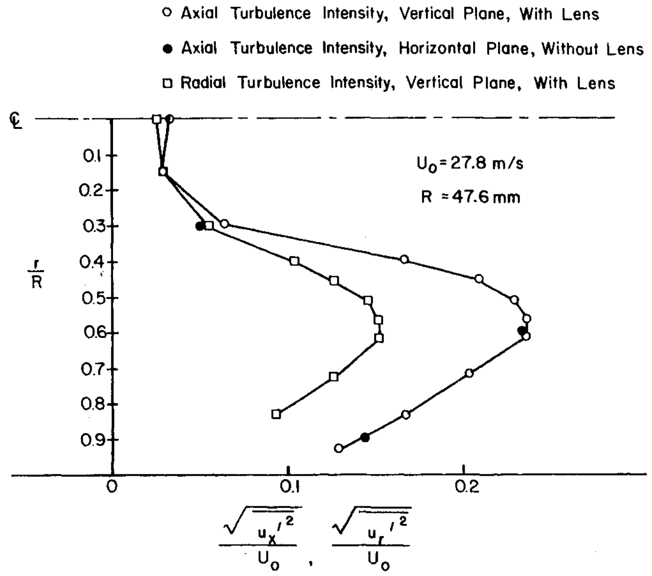

Fig. 7 Turbulence intensity profiles in the sudden expansion at $x/h = 4.0$.

Next, a full tube section was mounted on the bench with both a transmitting and receiving correction lens in place. The setup was used to check the receiving optics and is illustrated in Fig. 5. The location of the probe volume image was measured for various radial locations and with rotation of the input beams to insure that the LDV receiving optics would not require any major readjustment. The location of the probe volume image (where the photomultiplier tube pinhole would be located) remained stationary within approximately 0.38 mm in the x direction and 0.1 mm in the y direction. Thus, only minor readjustment of the pinhole location is needed to optimize signal quality after the system is moved to a new measurement position or rotated. It has been our experience that to maximize signal quality minor adjustments of the same magnitude are required when repositioning in a plane walled channel.

It is worthwhile to note that the beams exiting the tube are not symmetrical with the beams entering the tube because of the previous noted angular deviation of the input beam bisector. This caused no apparent optical problems, however, and the exiting beams were still focused at the proper location, even though the transmitting and receiving optics remained in the same relative position at all times. The separation of the correction lenses was changed for different measurement points along the tube radius as noted earlier, but no other adjustment of the optical elements was necessary.

Although the design program only employed ray traces in planes parallel and perpendicular to the tube axis, the lens was found to work satisfactorily for all angular orientations of the input beam plane. This was expected because the vector refraction equation (1) can be decomposed into three independent orthogonal scalar refraction equations. The scalar equations are independent and at each surface a skew ray can be resolved into orthogonal components and one equation applied to each component. If the angle between the beams is small the difference between the traces of the skew beams and the beams in the plane will be small.

Measurements with the Correction Lens

A series of measurements was made in a sudden pipe expansion flow to verify the performance of the lens under actual conditions. The LDV system and flow apparatus used in an earlier study[4] were employed for this purpose. Four separate measurements were made at each location: one in the axial direction, one in the radial direction, and one each at ± 45 with respect to the axial direction. The correction lenses were found to function properly out to a radius of $r/R \approx 0.83$ as predicted by the ray tracing program. At radii larger than this, individual measurements could still be made in specific directions, but readjustment of the optics was required for each component. In the axial direction, for example, measurements were made to $r/R > 0.95$ with good results. For the 90 deg orientation (radial direction) however, measurements could not be made beyond $r/R \approx 0.83$ because of the extreme bending due to the shallow intersection angle with the tube wall. Total internal reflection of the beams inside the tube wall was observed in several cases. At these large radial locations, the probe volumes oriented in the various directions were separated, as predicted by the ray tracing program and seen in the optical bench test.

Figures 6 and 7 show examples of data obtained in the flow downstream of a sudden pipe expansion with a diameter ratio of 1.875 and a downstream diameter of 95.2 mm. Axial mean velocities measured in the vertical plane (y axis in Fig. 1) using the correction lens are shown in Fig. 6 at four step heights downstream of the sudden expansion. Measurements made at four radial locations along the horizontal plane (x axis) in the conventional manner are shown for comparison. Agreement is seen to be excellent. Mean radial velocity data are also shown in Fig. 6. The values are small (note scale change) and there is thus more uncertainty in the data. However, the expected behavior is observed with near zero values close to the centerline and tube wall and peak values in the shear layer separating the primary flow from the recirculation zone. Obviously, it was not possible to measure the radial component along the horizontal plane for comparison.

Turbulence intensities obtained from the LDV data are shown in Fig. 7. Again, the axial component values on the x and y axes are in excellent agreement, indicating that the width of the measured velocity histograms as well as the mean values were not altered when the correction lens system was used. The relative turbulence intensity levels for the radial and axial components are similar to those that have been observed in two-dimensional step flows.[5]

Conclusions

This investigation showed that it is possible to use a rather simple correction lens system to negate many of the adverse effects of a cylindrical tube wall on laser velocimeter measurements. For a one-component LDV, the lens system reduces the realignment necessary when moving from one radial location to another and also reduces probe volume distortion and enlargement. For a two-component system, the lens system significantly reduces alignment problems associated with maintaining the two probe volumes at the same location. In both cases, the aberration of the probe volume image at the pinhole is minimized, thus enhancing signal quality.

It is probable that a more sophisticated correction lens design could be developed using multiple elements. However, the simple lens system described here should prove to be useful in many applications. Additional details regarding the design, fabrication, and use of the correction lens may be found in Ref. 6.

Acknowledgments

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